

Guidance to support the teaching of written calculations

We aim to ensure that by the end of year 6 as many children as possibly can will understand and use successfully compact written methods to carry out and record calculations that cannot do in their heads.

To enable children to move towards compact written methods with full understanding a step by step approach is taken. For each of the four operations children are first introduced to expanded methods that lead to a compact form of calculation. It is important that children feel secure and comfortable with each stage towards compact methods before they move on to the next. Children will progress through the stages of expanded calculations at different rates. It is far better that they can operate efficiently at any stage and with understanding than to move them on too quickly. Not all children will reach compact methods by the end of year 6.

Approach

The children need to approach any calculation by asking themselves the following questions:

- Can I do this in my head?
- Do I know the approximate size of the answer?
- If I cannot do it wholly in my head what do I need to write down in order to help me calculate the answer?
- Will the written method I know be helpful?

When considering more complex calculations, that they could not easily perform using pencil and paper procedures, they will also need to consider:

- Would it be more useful to use a calculator to work this out?

Whenever appropriate the children should do mental calculations. In order to support this approach calculations are always presented to children horizontally so they can make decisions as to the best way to solve the problem.

Addition and Subtraction

Criteria, which would indicate a child's readiness for formal written methods of addition and subtraction, would include:

- Knowledge of addition and subtraction facts to 20
- Understanding of place value and ability to partition numbers into hundreds, tens and units
- Understanding of commutative and associative laws of addition (though not of these terms)
- Ability to add at least three single digit numbers mentally
- Ability to add and subtract any pair of two digit numbers mentally
- Ability to explain mental strategies, orally and in writing

Progression in Addition

The following stages show a progression from informal methods for addition leading to a compact method.

Stage 1

Numbers of varying size using a number line

$$47 + 26$$

$$\begin{array}{ccc} 3 & 20 & 3 \end{array}$$

$$47 \quad 50 \qquad 60 \quad 73$$

Stage 2

Using Partitioning

$$47 + 76$$

$$70 + 6$$

$$40 + 7$$

$$110 + 13 = 123$$

Stage 3

Expanded vertical layout of a calculation showing the addition of the units and tens separately. Change to recording the least significant digit first

$$\begin{array}{r} 47 \\ + 76 \\ \hline 13 \\ 110 \\ \hline 123 \end{array}$$

Stage 4

Extend to bigger numbers recording the least significant number first.

$$\begin{array}{r} 368 \\ + 493 \\ \hline 11 \\ 150 \\ 700 \\ \hline 861 \end{array}$$

Stage 5

Introduce the compact layout alongside the expanded format. Introduce calculations that involve one carry

$$\begin{array}{r} 47 \\ + 76 \\ \hline 13 \\ 110 \\ \hline 123 \end{array} \qquad \begin{array}{r} 47 \\ + 76 \\ \hline 73 \\ 1 \end{array}$$

Stage 6

Introduce calculations that involve more than one 'carry'.

$$\begin{array}{r} 47 \\ + 76 \\ \hline 123 \\ 11 \end{array}$$

Stage 7

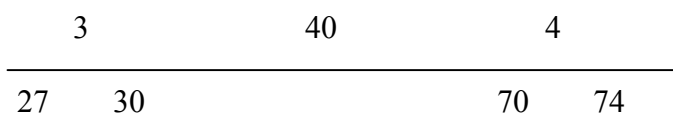
Use these layouts with bigger numbers and decimals. Children may need to revert to more expanded layouts initially.

Progression in Subtraction

Stage 1

Using a number line

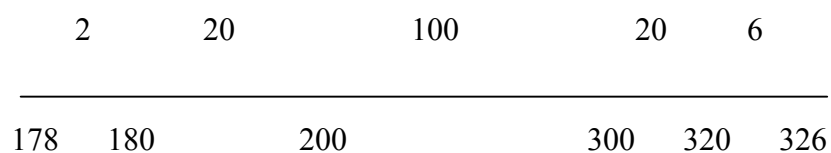
$$74 - 27$$



Stage 2

Using a number line with larger numbers and estimation

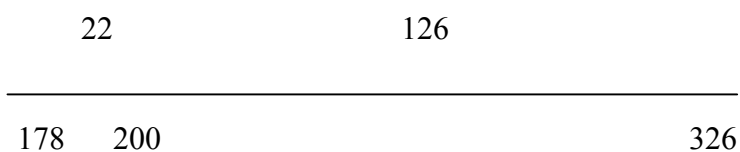
$$326 - 178$$



Stage 3

Using a number line with larger steps. Estimate first

$$326 - 178$$



Stage 4

Introduce expanded layout with partitioning, lining up vertically, and no decomposition

$$378 - 126$$

$$\begin{array}{r} 300 \qquad 70 \qquad 8 \\ 100 \qquad 20 \qquad 6 \\ \hline 200 \quad + \quad 50 \quad + \quad 2 = 252 \end{array}$$

Stage 5

Compact vertical layout with no decomposition, working from the least significant digit first in readiness for decomposition

$$\begin{array}{r} 378 \\ - 126 \\ \hline 252 \end{array}$$

Stage 6

Introduce decomposition with expanded vertical layout
 $326 - 178$

$$\begin{array}{r} 200 \qquad \overset{110}{\cancel{10}} \qquad 16 \\ \cancel{300} \qquad \cancel{20} \qquad \cancel{6} \\ 100 \qquad 70 \qquad 8 \\ \hline 199 \quad + \quad 40 \quad + \quad 8 \end{array}$$

Stage 7

Compact method with decomposition

$$\begin{array}{r} 2 \ 11 \ 1 \\ \cancel{3} \ \cancel{2} \ 6 \\ - \ 1 \ 7 \ 8 \\ \hline 1 \ 4 \ 8 \end{array}$$

Multiplication and Division

Criteria, which would indicate a child's readiness for formal written methods of multiplication and division, include:

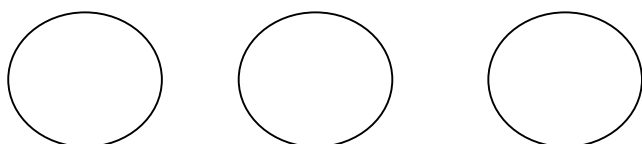
- Recall of multiplication and corresponding division facts for 2, 3, 4, 5 and 10 times tables
- Understanding of what happens when a number is multiplied by 0 or 1
- Understanding of place value
- Understanding of 0 as a place holder
- Ability to multiply two and three digit numbers mentally by 10 and 100
- Ability to approximate products and quotients using powers of 10
- Understanding of commutative, distributive and associative laws of multiplication (though not these terms)
- Ability to double and halve two digit numbers mentally
- Ability to use multiplication facts to derive mentally new multiplication facts
- Ability to explain mental strategies, orally and in writing

Progression in Multiplication

Stage 1

Using pictorial representation or written description

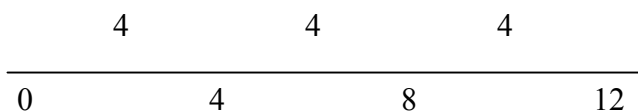
$$3 \times 4$$



Stage 2

Using a number line, showing repeated addition

$$3 \times 4$$



Stage 3

Using partitioning, a mental method, which forms a basis for future more compact methods.

$$38 \times 7 = (30 \times 7) + (8 \times 7)$$

$$= 210 + 56$$

$$= 266$$

Stage 4

The grid method which introduces partitioning using place value. The significant digit is multiplied first in order to get a sense of the size of the answer.

X	30	8	
7	210	56	266

Stage 5

Multiplication by a single digit, using expanded working in a vertical format. The least significant digit is recorded first. The children should be able to link this format with the grid method.

$$\begin{array}{r} 38 \\ \times 7 \\ \hline 56 \text{ (7 x 8)} \\ \underline{210} \text{ (7 x 30)} \\ \hline 266 \end{array}$$

Stage 6

Multiplication by a single digit, using vertical format and compact working.

$$\begin{array}{r} 38 \\ \times 7 \\ \hline 266 \\ 5 \end{array}$$

Stage 7

Extend grid method to larger numbers and multiplication by two digit numbers.

$$56 \times 27$$

x	50	6	
20	1000	120	1120
7	350	42	392
			1512

Stage 8

Multiplication by a two digit number using an expanded vertical format. Recording the least significant digit first.

$$\begin{array}{r} 56 \\ \times 27 \\ \hline 42 \quad (7 \times 6) \\ 350 \quad (7 \times 50) \\ 120 \quad (20 \times 6) \\ \underline{1000} \quad (20 \times 50) \\ \underline{1512} \end{array}$$

Stage 9

Compact method recording the least significant digit first. The children may need to use jottings to support multiplication.

$$\begin{array}{r} \text{71} \\ 56 \\ \times 27 \\ \hline 392 \quad (56 \times 7) \\ \underline{1120} \quad (56 \times 20) \\ \underline{1512} \\ 1 \end{array}$$

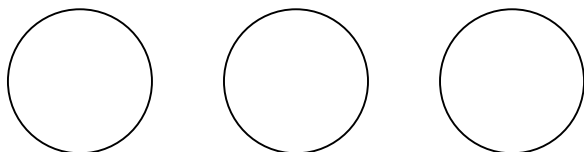
Progression in Division

Stage 1

Before introducing expanded methods division may be recorded in a number of ways to facilitate understanding.

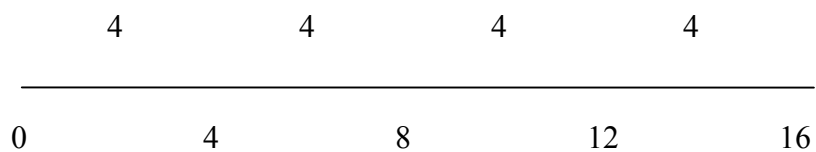
Sharing

$$12 \div 3$$



Recording on a number line

$$16 \div 4$$



Stage 2

Children are shown that division can be completed by repeated subtraction

$$25 \div 6$$

$$\begin{array}{r} 25 \\ -6 \\ 19 \\ -6 \\ 13 \\ -6 \\ 7 \\ -6 \\ 1 \end{array}$$

I took away 4 lots of 6 and had one left over.

Stage 3

Using known facts to arrive at an answer.

$$97 \div 9$$

We know $10 \times 9 = 90$ with a remainder of 7

$$97 \div 9 = 10 \text{ r } 7$$

Stage 4

Introduce vertical layout and the symbol for division $\overline{\hspace{1cm}}$

$$97 \div 9$$

$$\begin{array}{r} 9 \overline{)97} \\ \underline{90} \\ 7 \end{array}$$

Answer 10 r 7

Stage 5

Using vertical layout introduce larger numbers with the children subtracting in larger chunks provided they are comfortable with doing so. Answers should be estimated before the calculation.

$$\begin{array}{r} 6 \overline{)196} \\ \underline{-120} \text{ (6 x 20)} \\ 76 \\ \underline{-60} \text{ (6 x 10)} \\ 16 \\ \underline{-12} \text{ (6 x 2)} \\ 4 \end{array}$$

32 r 4

$$\begin{array}{r} 6 \overline{)196} \\ \underline{-180} \text{ (6 x 30)} \\ 16 \\ \underline{-12} \text{ (6 x 2)} \\ 4 \end{array}$$

Stage 6

Introduce the compact method, using zero as a place holder

$$\begin{array}{r} 32 \text{ r } 4 \\ 6 \overline{)196} \\ \underline{-180} \\ 16 \\ \underline{-12} \\ 4 \end{array}$$

Stage 7

Use the same methods for larger numbers and decimals

$$\begin{array}{r} 3 \cdot 25 \\ 6 \overline{)19 \cdot 50} \\ \underline{-18} \\ 15 \\ \underline{-12} \\ 30 \end{array}$$

C Murphy Jan-02